INTERRELATION BETWEEN THE PARAMETERS OF SOLIDIFICATION IN A CYLINDER WITH AXIAL AND RADIAL–AXIAL HEAT TRANSFER

F. I. Shved and D. A. Soskov

UDC 536.248.2

A method is described of calculating the temperature field in the crystallization zone (two-phase zone) which moves within a cylinder along the axis.

Let us assume an infinitely long cylindrical specimen of a multicomponent alloy which can be moved steadily in a temperature field where the two-phase liquid–solid zone bounded by two planes perpendicular to the cylinder axis will continuously move in the direction of this axis. There are two possibilities here: 1) heat flows into the two-phase zone through the liquid metal from which it is carried away into the solid along the specimen axis only (axial heat flow); 2) part of the heat flows into the two-phase region or is carried away from it through the lateral surface (radial–axial heat flow).

As basic parameters of the solidification process we will take the linear velocity of motion and the elongation of the two-phase zone, both together determining the length of time a metal remains in the two-phase state, i.e. the actual length of crystallization time.

We will mark out in the two-phase zone a small unit area perpendicular to the cylinder axis and we will analyze the change of heat flux through it due to displacement by an infinitesimal distance dx. The corresponding differential equation of heat transfer in a quasisteady process is

$$a^* \frac{\partial^2 t}{\partial x^2} = \frac{\partial t}{\partial \tau}. \tag{1}$$

For a single-phase medium not containing heat sources, $a = \lambda/\gamma c$. In our case latent heat of crystallization is given off in the two-phase zone and this brings about an increase in the apparent specific heat of the metal. We will assume that the latent heat is given off at a uniform rate throughout the crystallization range of temperatures, so that

$$c_0^* = c + \frac{L}{t_i - t_f}, \tag{a}$$

while

$$a_0^* = \frac{\lambda}{\gamma\left[c + L/(t_i - t_f)\right]}. \tag{b}$$

Inserting $\partial x = w\partial t$ in (1) and replacing a^* by a_0^*, we obtain for the axial heat flux:

$$a_0^* \frac{\partial^2 t}{\partial x^2} = w \frac{\partial t}{\partial x}. \tag{2}$$

Using conditions $x = 0$, $t = t_i$, $\partial t/\partial x = G_i$ and $x = \delta$, $t = t_f$, $\partial t/\partial x = G_f$, we can write

$$t = t_i + G_i \frac{a_0^*}{w}\left[\exp\left(\frac{w}{a_0^*}x\right) - 1\right], \tag{3}$$

Scientific-Research Institute of Metallurgy, Chelyabinsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 19, No. 1, pp. 106-109, July, 1970. Original article submitted June 27, 1969.

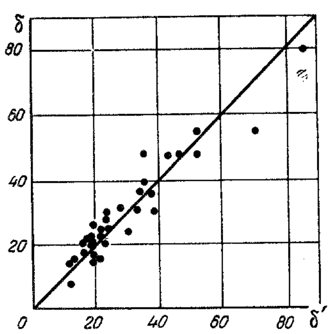

Fig. 1

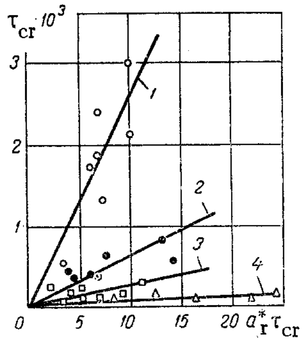

Fig. 2

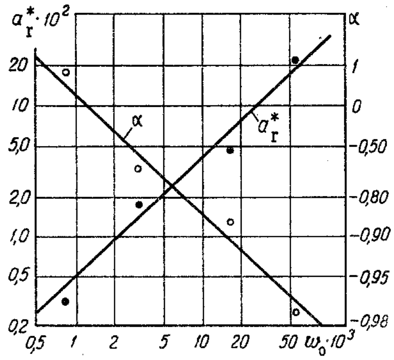

Fig. 3

Fig. 1. Comparison between the calculated (δ') and the actual (δ) width of a two-phase region (mm).

Fig. 2. The time a metal remains in the two-phase state τ_{cr} (sec), as a function of the calculated parameter $a_r^* \tau_{cr}$ (cm^2) and of the rate at which the specimen is lowered into a refrigerator. w_0: 1) $0.8 \cdot 10^{-3}$ cm/sec; 2) $3.0 \cdot 10^{-3}$ cm/sec; 3) $16.7 \cdot 10^{-3}$ cm/sec; 4) $53.3 \cdot 10^{-3}$ cm/sec.

Fig. 3. Effect of the lowering rate w_0 (cm/sec) on the apparent value of thermal diffusivity a_r^* (cm/sec) and on the coefficient $\alpha = Q/Q_0 = (a_0^*/a_r^*) - 1$.

$$\delta = \frac{t_i - t_f}{G_i - G_f} \ln \frac{G_f}{G_i}, \tag{4}$$

$$\tau_{cr} = \frac{\ln (G_f/G_i)}{(G_i - G_f)^2 a_0^*} (t_i - t_f)^2. \tag{5}$$

The appearance of a radial component of heat flux in the two-phase zone as well as the release of latent heat can be accounted for if the apparent values of specific heat and thermal diffusivity are modified accordingly:

$$c_r^* = c + \frac{L}{t_i - t_f} + \frac{Q}{t_i - t_f} \tag{c}$$

and

$$a_r^* = \lambda/\gamma \left(c + \frac{L}{t_i - t_f} + \frac{Q}{t_i - t_f} \right). \tag{d}$$

Replacing a_0^* by a_r^* in (3), (4), (5) will allow us to use these equations also for the radial–axial case. It follows from a transformation of (c) and (d) that

$$Q = (a_0^*/a_r^* - 1)[c(t_i - t_f) + L]. \tag{6}$$

Since $Q_0 = c(t_i - t_f) + L$ represents the total heat of crystallization, the coefficient $\alpha = (a_0^*/a_r^*) - 1$ characterizes the contribution of lateral heat flow during solidification. Through the lateral surface of the cylinder, heat is brought into the two-phase zone of the cylinder when $\alpha > 0$ ($Q > 0$) and heat is carried away from it when $\alpha < 0$. With $\alpha = 0$ ($a_r^* = a_0^*$) there is no heat flow across the lateral surface.

These relations were verified experimentally in a laboratory using a resistance furnace with a graphite heater and with a mechanism for lowering the molten steel specimen from the hot zone inside the furnace into a refrigerator at a rate which remained constant in each test but was varied from test to test between $0.5 \cdot 10^{-5}$ m/sec and $50 \cdot 10^{-5}$ m/sec. The metal temperature during crystallization was measured with tungsten–rhenium thermocouples at four locations along the specimen height. From the thermograms thus obtained, the solidification process parameters δ and τ_{cr} were found and then compared (Figs. 1, 2) with the values of δ' and $a_r^* \tau_{cr}$, which had been calculated by Eqs. (4) and (5). The slopes of the straight lines in Fig. 2 determine the magnitudes of coefficients a_r^* and α as functions of W_0 (Fig. 3).†

† a^* was calculated for steel with 0.5–0.7% carbon with $\lambda = 23.2$ J/m·sec·deg K, $\gamma = 7.5 \cdot 10^3$ kg/m^3, $c = 837$ J/kg·deg K, $L = 272$ kJ/kg, $t_i - t_f = 60$°K.

Here $\alpha \to (-1)$ corresponds to high values of w_0, while negligibly less heat flows axially than radially.

We observe an entirely definite correlation between the mode of heat flow and the structure of the metal: the dendrites are oriented parallel to the specimen axis when $\alpha > -0.9$, but they are oriented radially when $\alpha < -0.9$.

NOTATION

t	temperature;
x	distance along the cylinder axis;
τ	time;
a, a_0^*, a_r^*	thermal diffusivity, thermal diffusivity for a two-phase zone with axial heat flow and with radial–axial heat flow respectively;
λ	thermal conductivity;
c, c^*	specific heat, apparent specific heat;
γ	density;
L	latent heat of crystallization;
t_i, t_f	initial and final crystallization temperature;
G_i, G_f	temperature gradients at the boundary at the beginning and at the end of the solidification process;
Q	specific heat transfer through the lateral surface of a two-phase zone;
δ, δ'	actual and calculated elongation;
w	velocity of a moving two-phase zone;
τ_{cr}	duration of the two-phase state of a metal;
w_0	velocity at which the specimen is lowered into a refrigerator.